SUPPORTING TEACHERS' AND STUDENTS' KNOWLEDGE OF GEOMETRIC SIMILARITY

Nanette Seago
WestEd
nseago@wested.org

The main goal of the Learning and Teaching Geometry (LTG) project is to build professional development (PD) materials that provide opportunities for teachers to engage in learning about geometric similarity through the use of video cases, in which specific and increasingly complex mathematical ideas are presented within the dynamics of classroom practice. The central component of the LTG materials is the Foundation Module, which includes 30 hours of PD and is intended to provide teachers with a thorough grounding in key mathematical and pedagogical issues related to similarity. Field test data indicate that the Foundation Module supports gains in both teachers' and students' knowledge of similarity.

Keywords: Geometry, Mathematical Knowledge for Teaching, Teacher Education-Inservice/Professional Development, Teacher Knowledge

Introduction

Research has provided emerging evidence that certain characteristics of PD are important for influencing changes in classroom practice and supporting student achievement (Heck, Banilower, Weiss & Rosenberg, 2008; Sample McMeeking, Orsi & Cobb, 2012). For example, programs with a focus on subject matter knowledge and student learning in that subject are more likely to have an impact on student achievement than those focused on more generic topics (Cohen & Hill, 2000). In addition, numerous studies examining the effectiveness of mathematics and science PD have found that teachers who participated in activities that emphasized content knowledge, active learning, and coherence were more likely to report enhanced knowledge along with changes in their teaching practice compared to teachers who did not participate in such programs (Garet, Porter, Desimone, Birmin & Yoon, 2001).

There is strong evidence that best practices for mathematics PD include providing teachers with learning opportunities that are intensive in focus and extensive in duration (Garet et al., 2001) and that are "practice-based"—that is, they enable teachers to examine the mathematical skills and understanding that undergird their classroom curriculum, investigate students' mathematical thinking, and explore instructional practices that support student learning (Ball & Cohen, 1999; Cohen & Hill, 2000). By focusing on developing the understanding, skills, and dispositions that teachers use in daily practice, "practice-based" PD provides a meaningful context for teachers' learning.

Video can be an effective PD tool, as it brings the everyday work of teaching into a professional learning environment. Just as curriculum does not stand alone in classrooms, video does not stand alone in PD. Organizing PD around the study of videos does not, in itself, guarantee significant teacher learning any more than the use of technology in the classroom ensures that students will develop deep understandings. Like technology, videos are only tools; professional developers must help teacher educators use these tools to achieve specific learning goals. Without guidance, such as an analytic framework and explicit tasks, teachers watching videos rarely address subject matter content (Castro, Clark, Jacobs, & Givvin, 2005). Knowing how to observe teaching, what to look for and what to focus on, and how to talk about what one

sees are important skills to be learned (Castro, 2005). Learning these skills helps teachers to more skillfully "see" the subject matter in lessons, discriminate ways that learners comprehend subject matter; identify problematic features; assess student responses; detect, diagnose, and develop instructional responses to student errors; and generally gain more insights into their own practice (Givvin, 2005).

Theoretical Framework for the Learning and Teaching Geometry Project

The main goal of the Learning and Teaching Geometry (LTG) project is to build professional development materials that provide opportunities for teachers to expand their mathematical knowledge for teaching. The LTG PD materials help teachers to meet the challenges of teaching geometric similarity in accordance with the Common Core State Standards, which require defining and applying similarity based on geometric transformations—an approach that is likely to be new to most teachers and students.

The theoretical frame for the project is adapted from the work of Deborah Ball and colleagues (Cohen, Raudenbush & Ball, 2003) that incorporates research on both teaching and learning. As depicted in Figure 1, the content of the video case materials focus on the interactions between the teacher, the content (in this case, similarity tasks), and the students, within the context of an authentic classroom environment. The materials are designed to be used by a teacher educator who is faced with a similar set of relationships: the interactions between the teacher educator, the content (in this case, teaching and learning of geometric similarity), and the teachers with whom he or she works. The LTG Study is based on the premise that both classroom learning and professional learning opportunities are a function of these reciprocal relationships.

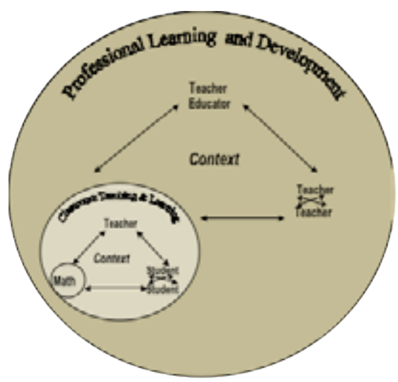

Figure 1: Theoretical Framework (Adapted from Cohen, Raudenbush & Ball, 2003)

The Learning and Teaching Geometry PD Materials

The LTG PD materials are intended for use in the professional development of mathematics teachers serving grades 5-10. Some of the videocases portray student thinking about particular concepts and some videocases portray pedagogical strategies and their impact on students' opportunities to learn (Seago, Driscoll & Jacobs, 2010). The materials follow a learning trajectory that is designed to enrich teachers' MKT as well as their ability to support students' understanding of similarity in alignment with the Common Core State Standards for mathematics. Specifically, they encourage teachers to gain a robust conception of similar figures as part of an infinite family that can be formed by applying one or more geometric transformations. Figure 2 illustrates the mathematical storyline in the Foundation Module, noting the central mathematical focus within and across sessions. The Foundation Module consists of

10, 3-hour sessions that are intended to provide teachers with a thorough grounding in key mathematical and pedagogical issues related to similarity.

Session 1	Session 2	Session 3	Session 4	Session 5	Session 6	Session 7	Session 8	Session 9	Session 10
A dynamic, transformational view of congruence	A dynamic, transformational view of similarity	Relationship between dilation and similarity	Properties of dilation	Preservation of angles & proportional lengths through dilation	Ratios within & between similar figures	Ratios within & between similar figures, Part 2	Connect-ions between similarity and slope & linearity	Area of similar figures	Closure and re-capping of big ideas
Defining Congruence and Similarity			Relationships and Attributes of Similar Figures				Connections		Closure

Figure 2: Mathematical Storyline of the LTG Foundation Module

Each session has at its core one or more digital video clips of a mathematics classroom. These clips are unedited segments selected from classroom footage of un-staged mathematics lessons, representing a range of grade levels, geographic locations and student populations across the United States. The clips offer a window into a variety of issues related to content, student thinking, and pedagogical moves. These clips typically represent a conceptual hurdle or portray some degree of mathematical confusion, based on the expectation that they are likely to provoke inquiry and discussion within the PD setting

Prior to watching a given video clip, teachers' grapple with the same mathematical task(s) the videotaped students tackled. Significant time is devoted to forecasting alternative solutions, comparing solutions, and anticipating student misconceptions. Typical conversations include consideration of: (1) the mathematical skills, procedures, and concepts entailed in the task; (2) the mathematical reasoning and solution strategies (correct and incorrect) that students are likely to apply to the task; (3) the affordances and constraints of different mathematical representations (e.g., pictures, tables, graphs, equations); and (4) instructional moves, such as questions and scaffolds, that might be helpful to support students' learning. Carefully designed focus questions help guide the viewing and analysis of the video clips, and provides teachers' support in unearthing student thinking. The questions and probes that accompany the LTG materials support a deep and substantive examination of classroom interactions.

Field Test Study of the LTG PD materials

Field Test Sites & Implementation

A field test of the LTG Foundation Module was conducted in 8 sites throughout the U.S. by a group of nine experienced facilitators personally recruited by the LTG staff. Most of the facilitators were state- or district-level middle and/or secondary mathematics coordinators, and two were university mathematics or education faculty. The field test was conducted over two academic years (2010-11, 2011-12) and used a treatment and comparison design in order to generate both formative and summative evaluation data related to impacts on teacher and student knowledge. Each facilitator recruited both treatment (87) and comparison (40) teachers. Typically, facilitators asked the treatment teachers to nominate comparison teachers from the same school and grade level.

Measures

To measure the impact of the LTG PD on teachers' mathematical knowledge for teaching in the domain of similarity, Horizon Research, Inc. (HRI) developed three instruments: a multiple-choice assessment, and two assessments embedded within the PD. HRI also developed a student assessment to measure the impact of teachers' participation in the LTG PD on their students' knowledge of similarity.

Teacher multiple-choice assessment. The multiple-choice teacher assessment includes 25 items targeting knowledge in five focus areas: (1) dilation; (2) properties of similarity; (3) ratios and proportions; (4) scaling; and (5) congruence transformations. The items were compiled and modified from released items used in state, national (NAEP), and international (TIMSS) assessment sources. About half of the items are purely content-based; the rest are set in teaching contexts that situate them in the work of teaching geometry. The items were reviewed by the LTG developers to validate them as accurate and appropriate to the content emphasis of the LTG PD materials. Pilot testing conducted as part of the project helped inform revisions to the instrument and demonstrated its sensitivity to teacher engagement with PD based on the LTG materials. The internal consistency of the full multiple-choice assessment was calculated using Cronbach's alpha as 0.81 for the pretest and 0.82 for the posttest.

Embedded assessments. The embedded assessments consist of two tasks that exist within the LTG Foundation Module: a mathematics task and a video analysis task. Both tasks address aspects of MKT, including subject matter content knowledge and pedagogical content knowledge (such as knowledge related to anticipating and interpreting student thinking about similarity). Each task was administered twice during the course of the intervention (first in an early session of the Foundation Module and again in a session near the end of the Foundation Module).

Student multiple-choice assessment. The student assessment contains 20 multiple-choice items and, like the LTG teacher assessment, targets content knowledge in five areas: (1) dilation; (2) properties of similarity; (3) ratios and proportions; (4) scaling; and (5) congruence transformations. Items were drawn from the same sources as the teacher assessment. In addition to review by LTG staff, HRI tested possible items through cognitive interviews with students in grades 7-9. Information gathered from these interviews helped to ensure the items were clear, plausible, and had content validity. Internal consistency reliability of the full student assessment was calculated using Cronbach's alpha as 0.71 for both the pretest and posttest.

Administration and Scoring of Measures

The three teacher knowledge assessments were each administered at two time points to 83 treatment teachers during implementation of the PD. Table 2 identifies the sessions in which the assessments were administered to treatment teachers. The assessments were also administered at two time points to 38 comparison teachers in separate meetings. Facilitators were asked to schedule the administration of assessments to the comparison teacher within the same timeframe as treatment teachers' participation in the PD in order to make data collection from these two groups consistent.

Results

Gains in teachers' geometry content knowledge. Analysis of the teachers' multiple choice assessment scores was conducted using repeated measures ANOVA to test for changes in percent correct scores over time. This approach controls for overall gains in the treatment and comparison teacher groups that may be attributed to learning from a year of teaching practice or from two administrations of the measure.

In the analysis, the main effect of group was non-significant, suggesting that the treatment and comparison teachers were initially comparable. A significant main effect of time (F = 21.09, $p < .05$) demonstrated that, on average, teachers improved from pre- to post-test. A significant time by group interaction term (F = 9.65, $p < .05$) indicated that the change in scores from pre- to post-test varied depending on participation in the treatment or comparison group. The treatment group demonstrated an average gain of 8.73 percentage points from pre- to post-test, whereas the comparison group demonstrated an average gain of 1.68 percentage points (see Table 1). In other words, teachers who experienced the LTG Foundation Module gained, on average, 7.05 more percentage points on their content assessment from pre- to post-test than teachers who did not experience the program. In pooled standard deviation units, the size of this effect was 0.39.

Table 1. Teachers' Geometry Assessment Scores

	Treatment Group (n = 83)		**Comparison Group (n = 38)**	
	Pre-test	**Post-test**	**Pre-test**	**Post-test**
Mean	**63.66**	**72.39 ***	**65.79**	**67.47**
(sd)	**(17.96)**	**(17.88)**	**(19.48)**	**(17.05)**

* A significant time by group interaction term indicated that treatment teachers demonstrated larger improvements in percent correct scores than comparison teachers (repeated-measures ANOVA; $p < .05$).

Gains in teachers' ability to apply their knowledge to classroom contexts. The mathematics task embedded assessment included three questions about similar rectangles. Questions 1 and 2 asked teachers to provide two different—but correct—methods to solve the same problem. Question 3 asked teachers to provide a method students might use that would lead to an incorrect solution. Scores for each question were analyzed using a Wilcoxon Signed Ranks Test for examining changes from pre- to post-test for each group. The False Discovery Rate adjustment for type I error was used to compensate for multiple comparisons.

Treatment teachers improved in their ability to provide a method, as shown in Table 2. For the treatment group, on average, there was a significant improvement in the scores on Questions 1 and 2 from pre- to post-test (p values $< .05$). The effect sizes for treatment group gains in scores on these questions were 0.30 and 0.32, respectively. There was not a significant change on Question 3 scores from pre- to post-assessment for treatment teachers. The comparison group, on average, did not demonstrate significant changes in score for any of the three questions.

Table 2. Teachers' Mathematics Task Embedded Assessment Scores

	Treatment Group (n = 82)				Comparison Group (n = 39)			
Question	Pre-test Mean (sd)	Post-test Mean (sd)	Ranks	Effect size	Pre-test Mean (sd)	Post-test Mean (sd)	Ranks	Effect size
1	2.56 (1.42)	3.17 (0.73)	33 + * 36 = 13 –	0.30	2.82 (1.45)	3.18 (1.07)	8 + 28 = 3 –	--
2	1.88 (1.53)	2.73 (1.38)	41 + * 29 = 12 –	0.32	1.95 (1.64)	2.23 (1.40)	16 + 14 = 9 –	--
3	2.73 (1.27)	2.78 (1.12)	27 + 31 = 24 –	--	2.85 (1.31)	2.77 (0.96)	14 + 15 = 10 –	--

[a] Ranks are reported with + indicating the number of teachers showing improvement from pre- to post-test, = indicating teachers with the same score on both administrations, and – indicating teachers with a decline.

- Significant improvement from pre- to post-test scores (Wilcoxon Signed Ranks Tests; p < .05)

The video analysis embedded assessment included three questions, prompting teachers to first interpret, and then apply, a student's approach to solving a similarity problem using dilation. For the comparison group, the Wilcoxon Signed Ranks Test did not reveal significant changes for any of the questions. Table 3 indicates that for the treatment group, on average, there was a significant improvement in the overall scores for all three questions, suggesting improved understanding of mathematical similarity in relation to dilation ($p < .05$ for all questions). The effect sizes for these questions were 0.32, 0.25, and 0.21 respectively.

Table 3. Teachers' Video Analysis Embedded Assessment Scores

	Treatment Group (n = 66) [a]				Comparison Group (n = 39)			
Question	Pre-test Mean (sd)	Post-test Mean (sd)	Ranks	Effect size	Pre-test Mean (sd)	Post-test Mean (sd)	Ranks	Effect size
1	3.89 (2.03)	5.02 (2.21)	39 + * 12 = 15 –	0.32	3.51 (2.61)	4.31 (2.14)	20 + 9 = 10 –	--
2	4.39 (2.55)	5.26 (2.09)	32 + * 16 = 18 –	0.25	3.87 (2.50)	4.38 (2.36)	19 + 5 = 15 –	--
3	2.00 (1.48)	2.56 (1.37)	29 + * 20 = 17 –	0.21	1.97 (1.50)	1.90 (1.50)	16 + 10 = 13 –	--

[a] Data from one field-test site was removed because participants discussed the video analysis task before providing written responses.
* Significant improvement from pre- to post-test scores (Wilcoxon Signed Ranks Tests; p < .05)

Gains in students' geometry content knowledge. A three-level hierarchical linear model was used to analyze data collected from students of treatment and comparison group teachers. This approach accounts for the repeated measure and the nested data structure of students in classrooms with the same teacher. A significant effect of time ($t = 5.69$, $p < .05$) indicated that across groups students' scores improved over the course of the year, as would be expected. Adding a time by group interaction term to the analysis revealed a significant difference ($t = 2.31$, $p < .05$) in gains between pre- and post-assessment scores for students of teachers who were engaged with LTG compared to students of teachers who were not involved in the PD (see Table 4). Average pre-post gains for students of treatment group teachers was 6.55 percentage points higher than gains for students of comparison group teachers. The size of this effect was 0.49 pooled standard deviation units.

Table 4. Students' Geometry Assessment Scores

	Treatment Group (n = 162) [a]		Comparison Group (n = 104) [a]	
	Pre-test	**Post-test**	**Pre-test**	**Post-test**
Mean	**36.42**	**45.28 ***	**42.69**	**45.00**
(sd)	**(17.14)**	**(17.15)**	**(17.63)**	**(18.97)**

[a] One treatment student's data and four comparison students' data were not included in the analysis because more than one-third of their item responses were left blank on one or both administrations of the assessment.

* A significant time by group interaction term indicated that students of treatment teachers' demonstrated larger improvements in percent correct scores than students of comparison teachers (repeated-measures ANOVA; $p < 0.05$).

The improvement among students of teachers engaged in LTG appears to have helped these students "catch up" to the students of teachers in the comparison group, whose estimated initial scores were, on average, more than six percentage points higher. In order to explore what changes in performance on the assessment accounted for this improvement, item-level results were examined by comparing the percent of students in the treatment and comparison groups that answered each item correctly on the pre- and post-tests. For 11 of the 20 items, the gain in the percent of students answering the item correctly favored the treatment group by 8 percentage points or more. None of the gains favored the comparison group by more than 5 percentage points. The 11 items with gains favoring the treatment group included all 3 items targeting dilation, 3 of the 4 items targeting congruence transformations, and both items targeting properties of similar figures. In the other two content areas, 2 of 6 items targeting scaling and 1 of 5 targeting ratios favored the treatment group. Although exploratory, these trends suggest that the treatment group students' improvements occurred in the content areas that are especially emphasized in the LTG Foundation Module and, prior to the release of the Common Core State Standards, have been less commonly addressed in middle school curricula (Confrey & Krupa, 2010; Heck, Weiss, & Pasley, 2011).

Discussion

Analyses from the field test study of the LTG PD materials offer evidence of the promise of the Foundation Module for achieving the intended teacher knowledge outcomes, including gains in geometry content knowledge along with the knowledge to apply understandings about content in mathematics instructional practice. These analyses used data from treatment and comparison teacher groups with efforts to ensure some degree of comparability across groups in terms of school and grade level. The analyses also controlled for teachers' initial status on the outcome measures. However, teachers were not randomly assigned to conditions, so factors other than the LTG PD may account in part for the impacts that were found. The field test also provides initial evidence that teachers' engagement in the LTG Foundation Module can lead to improvements in related student knowledge. An important limitation of the field test was the lack of systematic investigation of any changes that may have occurred in participating teachers' classroom geometry instruction, so the link to student outcomes, while promising, remains relatively tenuous.

The empirical evidence supporting the intervention has thus far been based on data in settings where the materials were used within the research and development process. The finalized version of the Foundation Module has not been tested, nor has the complete package of the LTG PD materials (which includes four Extension Modules) been used with any group of teachers at this point in time. Given the intervention's demonstrated potential, a critical next step is to study

the delivery of the finalized version of the intervention by trained, external facilitators, and carefully examining the impact on teachers' knowledge, changes in their instructional practices, and related improvements in their students' knowledge.

References

Borko, H., Koellner, K., Jacobs, J. & Seago, N. (2010). Using video representations of teaching in practice-based professional development programs. *Zentralblatt für Didaktik der Mathematik: International Reviews on Mathematical Education, 43*(1), 175-187.

Castro, A. (2005). A response to: Theory and practice question: Using video to support teacher learning. *AMTE Connections Summer 2005.*

Cohen, D.K. & Hill, H.C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record*, *102(2)*, 294-343.

Cohen, D.K., Raudenbush, S.W. & Ball, D.L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis, 25*(2), 119-142.

Desimone, L.M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38*(3), 181-199.

Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis, 24,* 81–112.

Castro, A., Clark, K., Jacobs J., & Givvin, K.B. (2005). Response to theory & practice question: Using video to support teacher learning. *AMTE Connections,* 14(3), 8-12

Confrey, J., & Krupa, E.E. (2010). *Curriculum design, development, and implementation in an era of Common Core State Standards: Summary report of a conference.* Columbia, MO: Center for the Study of Mathematics Curriculum.

Garet, M.S., Porter, A.C., Desimone, L., Birman, B.F., & Yoon, K.S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal, 38*(4), 915-945.

Givvin, K. (2005). A response to: Theory and practice question: Using video to support teacher learning. *AMTE Connections Summer 2005.*

Heck, D.J., Banilower, E.R., Weiss, I.R., & Rosenberg, S.L. (2008). Studying the effects of professional development: The case of the NSF's local systemic change through teacher enhancement initiative. *Journal for Research in Mathematics Education, 39*(2), 113-152.

Heck, D.J., Weiss, I.R., & Pasley, J.D. (2011). *A priority research agenda for understanding the influence of the common core state standards for mathematics: Technical report.* Chapel Hill, NC: Horizon Research, Inc.

Sample McMeeking, L.B., Orsi, R. & Cobb, R.B. (2012). Effects of a teacher professional development program on the mathematics achievement of middle school students. *Journal for Research in Mathematics Education*, *43(2)*, 159-181.

Seago, N., Driscoll, M., & Jacobs, J. (2010). Transforming middle school geometry: Professional development materials that support the teaching and learning of similarity. *Middle Grades Research Journal*, *5*(4), 199-211.